THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and twenty-seventh regular meeting of the Society was held in New York City on Saturday, February 24. The attendance at the two sessions included the following thirty members of the Society:

Professor G. A. Bliss, Professor C. L. Bouton, Professor Joseph Bowden, Dr. W. H. Bussey, Professor F. N. Cole, Dr. W. S. Dennett, Professor L. P. Eisenhart, Professor H. B. Fine, Mr. A. M. Hiltebeitel, Dr. Edward Kasner, Professor C. J. Keyser, Dr. G. H. Ling, Professor Max Mason, Mr. A. R. Maxson, Professor H. B. Mitchell, Professor Richard Morris, Professor W. F. Osgood, Miss I. M. Schottenfels, Professor Charlotte A. Scott, Professor C. S. Slichter, Dr. Clara E. Smith, Professor D. E. Smith, Professor P. F. Smith, Mr. A. W. Stamper, Dr. C. E. Stromquist, Professor H. D. Thompson, Professor E. J. Townsend, Professor Oswald Veblen, Miss E. C. Williams, Professor J. W. Young.

The President of the Society, Professor W. F. Osgood, occupied the chair. The Council announced the election of the following persons to membership in the Society: Mr. M. J. Babb, University of Pennsylvania; Mr. William Betz, East High School, Rochester, N. Y.; Mr. G. D. Birkhoff, University of Chicago; Mr. W. C. Breuke, Harvard University; Mr. B. E. Carter, Massachusetts Institute of Technology; Dr. H. L. Coar, University of Illinois; Miss Anna Johnson, Harvard University; Mr. W. D. Lambert, U. S. Coast Survey; Mr. W. A. Luby, Central High School, Kansas City, Mo.; President W. J. Milne, New York State Normal College; Professor Richard Morris, Rutgers College; Mr. W. J. Newlin, Harvard University; Miss R. A. Pesta, Wendell Phillips High School, Chicago, Ill.; Dr. H. B. Phillips, University of Cincinnati; Mr. A. R. Schweitzer, University of Chicago; Mr. C. G. Simpson, Michigan College of Mines; Mr. A. W. Stamper, Columbia University; Mr. F. C. Touton, Central High School, Kansas City, Mo.; Mr. M. O. Tripp, College of the City of New York. Ten applications for admission to the Society were received.

The date of the next annual meeting of the Society was fixed as Friday–Saturday, December 28–29. The summer meeting and colloquium will be held at Yale University, extending through the week September 3–8. Courses of colloquium lectures have already been arranged, and a preliminary announcement will be issued in May.

The following papers were read at the February meeting:

(1) Dr. W. H. BUSSEY: "On the tactical problem of Steiner."

(2) Miss I. M. SCHOTTENFELS: "On linear fractional transformations of functions of the complex variable $u + \epsilon v$, where $\epsilon^2 = 0$" (preliminary communication).

(3) Professor C. J. KEYSER: "On the linear complex of circle ranges in a plane."

(4) Professor E. B. WILSON: "Note on integrating factors."

(5) Miss R. L. CARSTENS: "A set of independent postulates for quaternions."

(6) Dr. W. B. FORD: "On the analytic extension of functions defined by double power series."

(7) Professor OSWALD VEBLEN: "Remark on a measure of categoricalness."

(8) Professor VIRGIL SNYDER: "Surfaces generated by conics cutting a twisted quartic curve and a line in the plane of the conic."

(9) Dr. CLARA E. SMITH: "Development of a function in terms of Bessel's functions (second paper)."

(10) Professor L. P. EISENHART: "Surfaces with the same spherical representation of their lines of curvature as spherical surfaces."

(11) Professor PAUL STÄCKEL: "Die kinematische Erzeugung von Minimalflächen (erste Abhandlung)."

(12) Professor OSKAR BOLZA: "A fifth necessary condition for a strong extremum of the integral

$$\int_{x_0}^{x_1} F(x, y, y')dx."$$

Miss Carstens's paper was communicated to the Society by Dr. Epsteen, Professor Stäckel's by Professor E. H. Moore. In the absence of the authors, the papers of Professor Wilson, Miss Carstens, Dr. Ford, Professor Snyder, Professor Stäckel and Professor Bolza were read by title. Abstracts of the papers

follow below; the abstracts are numbered to correspond to the titles in the list above.

1. The problem considered in Dr. Bussey's paper was proposed by Steiner in the *Journal für die reine und angewandte Mathematik*, volume 45, page 181. It has to do with the arrangement of N elements in triads, tetrads, pentads, etc. That part of the problem which relates to triads has been solved (see Encyclopédie des Sciences mathématiques, volume 1, page 80). The other parts are more difficult and have been made the object of but little study. By means of the properties of linear homogeneous equations in the Galois field of order 2, the author has completed the solution for the linear triple systems in $2^k - 1$ elements.

2. Miss Schottenfels treated the developments essential to the study of the linear fractional transformations of functions of the complex number $u + \epsilon v$, where $\epsilon^2 = 0$, including certain transformations such as $z' = z + \epsilon$, $z' = \mu z$.

3. Professor Keyser's paper, like that presented by him at the January meeting, deals with the circle range geometry of the plane. The range enjoys four degrees of indetermination. The ranges satisfying a single condition constitute a complex. A pencil of ranges is the ensemble of ranges having a common circle and lying in a circle congruence (totality of circles orthogonal to a given circle). The degree of a complex of ranges is the number of ranges common to it and an arbitrary pencil. When this number is 1, the complex is linear. The ranges of such a complex that have a given circle in common constitute a pencil, polar to the given circle. The ranges of the complex that lie in the congruence of a given pencil constitute a pencil, pole of the congruence. Hence a linear complex is a means of reciprocal transformation pairing the circles with the congruences of the plane and the ranges with the ranges. The ranges of a pair are conjugates. The circles of a range correspond to the congruences of the conjugate range, and reciprocally. The anharmonic ratio of any four circles of a range is equal to that of the corresponding congruences of the conjugate range, and reciprocally. To any configuration of circles, ranges and congruences corresponds a (reciprocal) configuration of (the polar) congruences, ranges and circles. Every range having a circle in common with each of two con-

jugate ranges belongs to the complex. If a range r of the complex has a circle in common with a given range r_1, r contains a circle of the conjugate range r_1' of r_1. According as two conjugate ranges contain or do not contain a common circle (or congruence) they do or do not belong to the complex. If they do, they coincide. Three independent ranges r_1, r_2, r_3 determine an infinity of ranges each intersecting each of the given r's in a circle. The ranges so determined are readily constructible. They constitute a system S of generating ranges of a quadric configuration of ranges, analogous to the simple hyperboloid of space. The second system S' of generating ranges is determined by any three ranges of S. If the ranges of S belong to a complex C, they are each self-conjugate as to C. Then no range of S is in C but the conjugate of every range of S is in S'. The radical axes of the ranges in S envelope a conic; similarly for S', and the conics coincide. In case of the analogue of the hyperbolic paraboloid, the conic degenerates into a pair of pencils of lines.

4. Professor Wilson gave an elementary proof of the relation $MV X_1^2 + X_2^2 + \cdots + X_n^2 = dF/dn$ between the coefficients X_i, the integrating factor M, and the solution F of an integrable total differential equation $\sum X_i dx_i = 0$. This relation was then used to discuss, geometrically and somewhat more in detail than is usual, the matter of singular solutions, limiting solutions and the factors of $M^{-1} = 0$. The paper is to appear in the *Annals of Mathematics*.

5. In this paper Miss Carstens defines Hamilton's quaternions by a set of independent postulates based on Professor Dickson postulates for hypercomplex number systems (*Transactions*, volume 6, No. 3, pages 344–348; 1905).

6. Dr. Ford's paper considers the functions $f(x, y)$ defined by the double power series

$$\sum_0^\infty \sum_0^\infty a(m, n) x^m y^n. \tag{1}$$

It is shown that under certain conditions for the function $a(m, n)$ the function $f(x, y)$ may be extended analytically outside the circles of convergence of the series (1) and throughout the entire x, y planes with the exception of the cuts 0 to $+\infty$ along

the real x and y axes. The paper will appear in the *Transactions*.

7. Professor Veblen's note appeared in the March BULLETIN as the last part of his review of Huntington's Types of Serial Order.

8. If a line and a twisted quartic be given, and a correlation between the planes and points of the line be established, then a conic is uniquely fixed by the four points on the curve and the point associated with the plane. The surfaces studied by Professor Snyder are described by the conic when the plane turns about the line, which was chosen in various ways with regard to the curve.

9. Schlömilch determined the coefficients of his development of an arbitrary function $f(x)$ in terms of $J_0(x)$ by applying Abel's relation

$$\frac{2u}{\pi}\int_0^1 dt \int_0^1 \frac{f'(\lambda u t)\lambda d\lambda}{\sqrt{1-t^2}\sqrt{1-\lambda^2}} = f(u) - f(0),$$

to an auxiliary function in terms of which $f(x)$ could be expressed. The justification of this method involves certain conditions on the second derivatives of $f(x)$. Miss Smith showed that this unnecessary restriction can be removed by applying Abel's relation directly to $f(x)$.

Another development in terms of $J_0(x)$, whose coefficients differ slightly in form from those of Schlömilch, can be justified under slightly more general conditions. In simple cases the two developments are identical. Analogous to this second development is one in terms of $J_1(x)$, which is much more easily justified than that which Schlömilch obtained by termwise differentiation of the series in $J_0(x)$. These also are in many cases identical.

10. In several previous papers Professor Eisenhart has considered the surfaces with the same spherical representation of their lines of curvature as pseudospherical surfaces, called for convenience A-surfaces. Now he discusses the case where the representation is that of spherical surfaces, that is, surfaces with constant positive curvature, pointing out his previous results which have a significance in the present case and estab-

lishing theorems which of necessity have no analogues in the other theory. There exists an imaginary transformation of one of the surfaces into another of the same kind, which is similar to the generalized Bäcklund transformations of A-surfaces. Pairs of these transformations can be found which when applied successively to a real surface yield a new real surface of the same kind. Bonnet showed that these surfaces go in pairs — the members of a pair being applicable with correspondence of the lines of curvature; and, moreover, these surfaces are the only ones applicable in this manner. On this account we call them surfaces of Bonnet. The knowledge of a transformation of such a surface enables one to find by algebraic processes the surface of Bonnet applicable to the given one. By means of the above-mentioned transformations a pair of real applicable surfaces of Bonnet can be transformed into a new pair. There exist a large number of surfaces whose coördinates are expressed in forms similar to those for surfaces of Bonnet and the surfaces analogous to the latter considered elsewhere by the author.

11. A curved surface is said to have a kinematic generation if it is generated by a rigid curve moving according to a given law. To surfaces generated kinematically belong the minimal surfaces whose generators are imaginary curves with vanishing line element. Professor Stäckel proposes to publish in a series of articles the results of investigations which he is making on those exceptional minimal surfaces that have more than one kinematic generation. The first paper, which will appear in the *Transactions*, is concerned with the minimal surfaces that can be determined in more than one way as translation surfaces. A new and direct proof is given of Lie's theorem that the Scherk's surfaces are the only minimal surfaces which can be generated by the translation of curves with non-vanishing line element. For, if it be required that a surface of translation be at the same time a minimal surface, then the gaussian parameter representation gives a functional equation for three functions of u and three functions of v. A complete solution for this equation is determined. If the generating curve is plane, the Scherk's surfaces are obtained immediately; but if it is a twisted curve in space, then the solution of the given functional equation reduces to the integration of the system of ordinary differential equations

$$\frac{hg' - gh'}{f^2 + g^2 + h^2} = s_{11}f + s_{12}g + s_{13}h,$$

$$\frac{fh' - hf'}{f^2 + g^2 + h^2} = s_{21}f + s_{22}g + s_{23}h,$$

$$\frac{gf' - fg'}{f^2 + g^2 + h^2} = s_{31}f + s_{32}g + s_{33}h,$$

in which f, g, h are functions of u and f', g', h' their derivatives, while $s_{11}, \cdots, s_{33}$ are constants. These equations also give the Scherk's surfaces, which admit an infinite number of generations in the way described; moreover the ordinary helicoidal surfaces appear as special cases of this result.

12. It is well known that the conditions of Euler, Legendre, Jacobi, and Weierstrass are not sufficient for a strong extremum of the integral

$$J = \int_{x_0}^{x_1} F(x, y, y')dx.$$

In Professor Bolza's paper a fifth necessary condition is established. The paper will be published in the current volume of the *Transactions*.

F. N. COLE,
Secretary.

THE FIFTY-FIFTH ANNUAL MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE American Association for the Advancement of Science held its fifty-fifth annual meeting in New Orleans, the sessions continuing from December 28, 1905, to January 3, 1906.

The president of the meeting was Professor Calvin M. Woodward of St. Louis. Dr. L. O. Howard, Washington, D. C., is the permanent secretary of the Association. The enrollment was small, reaching a total of only 233, and the programmes of many of the sections were unusually brief, but the meeting as a whole can by no means be considered unsuccessful. It is believed by many that, though the attendance may always be small, one of the most important purposes of the organization — the stimu-

lation of popular interest in science — will be better achieved by meeting more frequently in the remoter cities of the country. It is to be noted, however, that few of the affiliated societies find it advisable to meet at a great distance from the more populous educational centers. Thus, another important function of the Association — the coördination of scientific interests — is certain to be suspended as often as meetings at distant points are undertaken. It should in any case be said, with reference to the New Orleans meeting, that the welcome accorded the Association was remarkably cordial.

The address of the retiring president, Professor W. G. Farlow, of Cambridge, Mass., was given at Sophie Newcomb College on the evening of December 29, the subject being 'The popular conception of the scientific man at the present day.' It has been published in full in *Science* for January 5 of the current year.

The meetings of Section A (mathematics and astronomy), as well as those of the other sections, were held in the buildings of Tulane University. The officers of the section were: vice-president, W. S. Eichelberger; secretary, L. G. Weld; councilor, C. S. Howe; member of the general committee, G. B. Halsted; press secretary, the secretary of the section; sectional committee, Alexander Ziwet, J. R. Eastman, Ormond Stone, E. B. Frost, E. O. Lovett, Harris Hancock, together with the vice-president and the secretary of the section. In the absence of the vice-president the retiring vice-president, Professor Ziwet, acted as chairman. The following mathematicians and astronomers were, upon nomination by the sectional committee, elected by the council to fellowship in the Association: E. W. Brown, W. A. Granville, Harris Hancock, M. W. Haskell, E. J. Townsend, Irving Stringham, Paul Wernicke.

The next annual meeting of the Association will be convened in New York City, on Thursday, December 27, 1906. In addition to this meeting it was decided by the general committee to hold a special summer meeting at Ithaca, New York, to close on or before July 3. There will be no presidential or vice-presidential addresses at the summer meeting. All officers elected at the New Orleans meeting will hold over to the close of the New York meeting. Dr. W. H. Welch of Baltimore, Md., was elected president of the New York and Ithaca meetings, Dr. Edward Kasner of New York City will be vice-president of section A. The present secretary of the section is to

continue in office. Chicago was recommended as the place of meeting in 1907.

The address of the retiring vice-president of section A, Professor Alexander Ziwet, on 'The relation of mechanics to physics,' was presented on the afternoon of December 29. It has already been published in full in *Science* for January 12. At the regular programme meeting of the section, held on December 30, eleven papers were presented. The titles of these are given below, with abstracts of such as deal with purely mathematical subjects.

(1) Dr. O. E. GLENN: "On the groups of order $p^m q^n$ having abelian subgroups H_{p^m} of type $[n, n, \cdots, n]$."

(2) Professor G. B. HALSTED: "A new straight in non-euclidean geometry."

(3) Professor HARRIS HANCOCK: "A chapter in the present state of development of the elliptic functions."

(4) Mr. H. B. HEDRICK: "A catalogue of 1607 zodiacal stars for the epochs 1900 and 1920, reduced to an absolute system."

(5) Dr. EDWARD KASNER: "A class of central forces."

(6) Professor F. H. LOUD: "Solar photographs."

(7) Professor G. A. MILLER: "The groups of order p^m which contain exactly p cyclic subgroups of order p^{α}."

(8) Mr. J. J. QUINN: "Inversion and inversors."

(9) Professor DAVID TODD: "Observations of the total solar eclipse of 1905, August 30, at Tripoli, Barbary."

(10) Professor DAVID TODD and Mr. R. A. BAKER: "Computed traces and totality durations of the total eclipses of the twentieth century."

(11) Professor L. G. WELD: "A possible extension of the theory of envelopes."

1. This paper is supplementary to one presented by the author at the Philadelphia meeting of the Association, in which the case $n = 1$ was discussed. The defining relations of all groups described in the title are tabulated, and their properties discussed in relation to the properties of the Galois field $GF[p^{mn}]$ determined by the automorph of the subgroup H. It is found that all of the groups in question are members of a general family and have one general set of defining relations.

2. The paper sets forth the discovery that, in Riemann non-euclidean geometry, the six mid-points of the parts of the

six rays from the vertices of any triangle obtained by prolonging the sides are costraight. This is a new and noteworthy straight associated with every triangle. The theorem is demonstrated, and then interpreted in ordinary euclidean space.

3. This paper is an attempt to show that practically all (American and European) writers on the elliptic functions have been giving too much emphasis to certain parts of Weierstrass's theory, while they have neglected many of the lines of thought which Weierstrass himself considered fundamental.

It is shown that the so-called Weierstrass normal form is not due to Weierstrass. The introduction of new functions gives a different aspect to the presentation of the elliptic functions although little that is new has been added thereby to the theory itself. Weierstrass's great work lies in a somewhat different direction. With him the problem of determining all analytic functions which have algebraic addition theorems is the leading idea.

The paper also cites several fundamental theorems of Hermite and indicates some of the characteristics of Riemann's theory.

4. This paper will be published in the Astronomical Papers of the American Ephemeris, volume 8, part 3.

5. There exists no field of force in which a particle started from an arbitrary position with arbitrary velocity will describe a circular path. In the case of a central force the only possible circular trajectories are, in general, those whose centers are at the origin of force. If, however, the force varies according to a function of the form $br(r^2 - a)^{-3}$, then a quadruple infinity of the trajectories are circular. In the simplest case, arising when a vanishes, the force varies inversely as the fifth power of the distance, and the circles all pass through the origin. In the general case they are orthogonal or diametral to a fixed sphere.

7. The main theorems proved in this paper may be stated as follows: If a group of order p^m, p being any odd prime, contains exactly p cyclic subgroups of order p^α, $\alpha > 2$, it contains exactly p cyclic subgroups of every order which exceeds p and divides p^{m-1}. Hence it is one of the two non-cyclic groups of order p^m which contain operators of order p^{m-1}. When $\alpha = 2$

and $p > 3$ the theorem is still true. In fact, the only possible exception occurs when $\alpha = 2$, $p = 3$ and $m = 4$. In this special case there are three groups which contain exactly p cyclic subgroups of order p^α.

When $p = 2$ the preceding theorem is replaced by the following: If a group of order 2^m contains exactly two cyclic subgroups of order 2^α, $\alpha > 2$, it cannot contain more than two cyclic subgroups of any higher order. If a group of order 2^m contains exactly two cyclic subgroups of order 2^β but does not contain any cyclic subgroup of order $2^{\beta+1}$, then m cannot exceed $2^{\beta+2}$. These theorems involve the fundamental properties of all possible groups whose order is a power of any prime p and which involve exactly p cyclic subgroups of any given order p^α. From a well known theorem it follows that α is not unity, but it can have every other possible value less than m.

8. In this paper are presented two new theorems relating to inversion, besides an explanation of the construction of certain linkages exhibiting the operation of inversion.

11. (*a*) In the equation $f(\alpha, x, y) = 0$, representing a family of loci, by giving to α, first an increment and then a corresponding decrement, each of magnitude $\Delta\alpha$, solving the resulting equations for the coördinates of the point of intersection and, finally, letting $\Delta\alpha \doteq 0$, there will be obtained $x' = \phi(\alpha)$, $y' = \psi(\alpha)$. These equations define a point of the envelope of the given family of loci and eliminating α between them gives $F(x', y') = 0$, the equation of the envelope.

The point (x', y'), determined as above, may be called the tracing point of the locus; that is, the point which, for the moment, is tracing the envelope. It was shown in the paper, by way of illustration, that the tracing point for the envelope of the family of ellipses

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad \alpha + \beta = c$$

is the Fagnani point.

(*b*) The inverse of the above notion was next developed with reference to the right line, viz: A point on the line

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

being assigned at will, to find the functional relation between the intercepts, $\Phi(\alpha, \beta) = 0$ (*i. e.*, the law governing the motion of the line), in order that the given point may trace an envelope and, finally, to obtain the equation of the envelope. The required relation is given by either of the differential equations

$$x' = \phi(\alpha, \beta) = \frac{\alpha^2}{(\alpha - \beta)d\alpha/d\beta}, \quad y' = \psi(\alpha, \beta) = \frac{\beta^2}{(\beta - \alpha)d\beta/d\alpha}.$$

In general both equations will be needed in order to determine the constants of integration. Having thus obtained the function Φ, which is, in effect, the tangential equation of the envelope, the equation in rectangular coordinates readily follows.

Several examples applying the principles were presented and its application to other families of loci was suggested as a promising field of investigation for the amateur mathematician.

LAENAS GIFFORD WELD,
Secretary.

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A PROOF OF THE FUNDAMENTAL THEOREM OF ANALYSIS SITUS.

BY PROFESSOR G. A. BLISS.

(Read before the American Mathematical Society, December 28, 1905.)

THE theorem that a Jordan curve divides the plane into two regions, an interior and an exterior, has in recent years received much attention. The proofs which have been given may be roughly divided into two classes, those in which the object has been to prove the theorem with the fewest possible hypotheses on the curve,* and those in which generality has to a certain extent been sacrificed for simplicity.† The following proof belongs to the second class. In § 1 it is assumed that the curve considered is continuous and has a continuously turning tangent at every point ; but in § 3, by extending the proof of one of the auxiliary theorems, curves with a finite number

* Veblen, *Transactions Amer. Math. Society*, vol. 6 (1905), p. 83.

† Ames, *Amer. Jour. of Math.*, vol. 27 (1905), p. 353. Bliss, BULLETIN, vol. 10 (1904), p. 398. For further references, see the paper by Ames.

of corner points and singularities are also included. The proof is presented here because it seems shorter and simpler than those heretofore given.

§ 1. *Hypotheses on the Curve.*

The curve to be considered is for the present supposed to be closed, but otherwise non-intersecting, continuous with a continuously turning tangent at every point, and to have no singular points. If its equations are given in the form

$$C: \qquad x = \phi(t), \quad y = \psi(t),$$

these conditions may be expressed analytically as follows:

(a) The functions ϕ and ψ are periodic with period ω, but the points (x, y) defined by two different parameter values t, t' are distinct unless t and t' differ by a multiple of ω.

(b) ϕ and ψ are continuous for all values of t.

(c) The derivatives ϕ' and ψ' are continuous for all values of t.

(d) $\phi'^2 + \psi'^2 > 0$ for all values of t.

Let (ξ, η) be a point of the curve C defined by a parameter value τ. On account of (d) and (c) one of the derivatives, say ϕ', is different from zero in an interval $[\tau - \delta, \tau + \delta]$, and as t traverses the interval x varies monotonically between two values ξ_1 and ξ_2. Under these circumstances t is similarly a monotonic continuous function of x in the interval $[\xi_1, \xi_2]$, and this function substituted for t in $y = \phi(t)$ gives an equation in the form $y = f(x)$.

For any point (ξ, η) of the curve C there exists an interval $[\xi_1, \xi_2]$ including the value ξ, such that all the points of C near (ξ, η) satisfy an equation

$$(1) \qquad y = f(x), \quad (\xi_1 \leqq x \leqq \xi_2);$$

or else there exists an interval $[\eta_1, \eta_2]$ including η, such that

$$x = g(y), \quad (\eta_1 \leqq y \leqq \eta_2).$$

The function f, or g, is single-valued and continuous.

With the help of the last statements the following important auxiliary theorem can be readily proved:

AUXILIARY THEOREM I. *At any point (ξ, η) of the curve C, a rectangle with its center at (ξ, η) and sides parallel to the axes*

can be constructed, in such a way that it is intersected only twice and divided into two continua by the curve.

To fix the ideas consider a point near which the curve has the form (1). A square of side 2ϵ with its center at (ξ, η) will be intersected only by the arc (1) if the half-diagonal $\epsilon\sqrt{2}$ is taken less than the shortest distance from (ξ, η) to other parts of the curve. If another positive constant δ is now taken so that in the interval $[\xi - \delta, \xi + \delta]$ the absolute value of $f(x) - \eta$ is less than ϵ, then the rectangle whose sides are $x = \xi \pm \delta$, $y = \eta \pm \epsilon$ is of the desired kind. In one of the two continua $f(x) - \eta$ is positive, in the other negative.

§ 2. *Existence of at most Two Continua.*

The curve C divides the plane into continua R_i, perhaps infinite in number, whose only boundary points are points of the curve. With the help of the auxiliary theorem of § 1 it may be shown that the number of continua is at most two.

AUXILIARY THEOREM II. *Each continuum R_i has every point of the curve C as a boundary point. On the other hand the (x, y)-points near any point of C belong to at most two continua, so there are in all at most two.*

It is evident that any continuum R_i must have at least one boundary point (ξ, η) on the curve C. It follows that R_i must include one of the continua into which the rectangle about (ξ, η) is divided according to the first auxiliary theorem, and must therefore have all the points of C near (ξ, η) as boundary points. Consider now the largest interval $\tau \leqq t < T$ $(T \leqq \tau + \omega)$ of parameter values all of which define boundary points of R_i. The upper boundary T can not be less than $\tau + \omega$, for the point (x, y) defined by it is a limit point of boundary points, and therefore is itself a boundary point of R_i. With the help of Auxiliary Theorem I it follows as above that all the parameter values near T also define boundary points of R_i, and the interval $\tau \leqq t < T$ with $T < \tau + \omega$ could not be the largest.

§ 3. *Existence of at least Two Continua.*

The existence of at least two contiuua is shown by means of a function $N(a, b)$ which is constant as long as (a, b) remains in one of the continua R_i, but which takes at least two different values at points of the plane not on the curve C. This function is the integral

$$(2) \qquad N(a, b) = \int_t^{t+\omega} \frac{(\phi - a)\psi' - (\psi - b)\phi'}{r^2}\, dt$$

$$[r^2 = (\phi - a)^2 + (\psi - b)^2]$$

whose primitive is the function u defined by the equations

$$\cos u = \frac{\phi - a}{r}, \quad \sin u = \frac{\psi - b}{r}.$$

It is evident from (2) that $N(a, b)$ is a continuous function of a and b when (a, b) is not on the curve C, and on account of the periodicity of ϕ and ψ its value is always some multiple of 2π. Since any two points in the same continuum R_i can be joined by a curve along which N varies continuously, the values of N at points in the same region must be always equal to the same multiple of 2π.*

In order to show that $N(a, b)$ has at least two different values in the plane, consider again a point (ξ, η) near which the curve C has the form (1), and let $h > 0$ be so chosen that the only point of C on the ordinate $x = \xi$ between $\eta - h$ and $\eta + h$ is (ξ, η). Denote by N, N_1, u, u_1 and r, r_1 the values of $N(a, b)$, the angles, and the radii corresponding to the two points $a = \xi$, $b = \eta \pm h$. The difference $N - N_1$ is an integral whose primitive is the function $u - u_1$ defined by the equations

$$\cos (u - u_1) = \frac{(\phi - \xi)^2 + (\psi - \eta)^2 - h^2}{rr_1},$$

$$\sin (u - u_1) = -\frac{2h(\phi - \xi)}{rr_1}.$$

On account of the periodicity of ϕ and ψ the values of $u - u_1$ at t and $t + \omega$ differ by a multiple of 2π. Furthermore, as t varies through an interval ω, $u - u_1$ passes once and only once through the value π. For when $u - u_1 = \pi$,

$$\phi - \xi = 0, \quad \frac{(\psi - \eta)^2 - h^2}{rr_1} = -1, \quad \therefore\ |\psi - \eta| < h,$$

* The quotient $N/2\pi$ has been called by Ames the "order" of the point (a, b); loc. cit., p. 353. The proof given in this section is similar to his, and to one about to be published by Osgood in his Lehrbuch der Funktionentheorie, vol. 1, p. 136.

and the point (ϕ, ψ) must be the point (ξ, η), the only point of the curve C on the ordinate $x = \xi$ between $\eta - h$ and $\eta + h$. That $u - u_1$ actually passes through the value π follows because $\phi - \xi$ and consequently $\sin(u - u_1)$ change sign when t passes through the value τ. It is evident, then, that the values of $u - u_1$ at t and $t + \omega$ differ by $\pm 2\pi$, and hence

$$N - N_1 = \pm 2\pi.$$

AUXILIARY THEOREM III. *The function $N(a, b)$ is a constant in any continuum R_i and takes at least two different values at points (a, b) not on the curve C. There exist therefore at least two continua R_i.*

The theorem which was the object of the present paper follows at once from the last two auxiliary theorems:

PRINCIPAL THEOREM. *Any Jordan curve having the properties stated in §1 divides the plane into two continua, an interior and an exterior.*

§4. *Extension to Curves with Corners and Singular Points.*

The proof given in the preceding sections can be extended without much difficulty to curves having a finite number of points where the conditions (*c*) and (*d*) of §1 are not both satisfied. In order to effect the extension it is only necessary to show that a rectangle with the properties described in Auxiliary Theorem I can be constructed at the exceptional points as well as at the others. The further steps in the proof remain the same.

The exceptional points admitted are points where conditions (c) and (d) of §1 do not both hold true, but it is supposed that the direction of the tangent defined by the equations

$$\cos a = \frac{\phi'}{\sqrt{\phi'^2 + \psi'^2}}, \quad \sin a = \frac{\psi'}{\sqrt{\phi'^2 + \psi'^2}},$$

approaches definite limiting values as the parameter t approaches, from either direction, the value τ defining the exceptional point. The two limits corresponding to the two directions of approach are not necessarily the same.

Such points include corner points where all the conditions of §1 are satisfied except that ϕ' and ψ' are discontinuous, and

singular points where both ϕ' and ψ' are zero provided that higher derivatives of ϕ and ψ exist which do not vanish simultaneously at the value τ.

Suppose that the X-axis is taken so as not to be perpendicular to any of the limiting directions of the tangent at exceptional points. Then there is an interval $\tau - \delta_1 \leqq t < \tau$ in which $\cos \alpha$ and ϕ' are different from zero, although at $t = \tau$ the derivative ϕ' may vanish. In such an interval x is a monotonic function of t, varying from ξ_1 to ξ. Conversely t, and therefore also y, is a single-valued continuous function of x in the interval $[\xi_1, \xi]$. In a similar way y may be expressed as a function of x in an interval $[\xi, \xi_2]$ corresponding to parameter values between τ and $\tau + \delta_2$. Let these two functions be denoted by $y = f_1(x)$ and $y = f_2(x)$ respectively.

There are two cases to be considered. If ξ_1 and ξ_2 include the value ξ between them, then y is a single-valued continuous function of x in the whole interval $[\xi_1, \xi_2]$ and the rectangle can be constructed exactly as in § 1. If ξ_1 and ξ_2 are both on the same side of ξ, then the intervals $[\xi_1, \xi]$ and $[\xi, \xi_2]$ overlap, but $f_1(x)$ is always different from $f_2(x)$ in the common interval. The construction of the rectangle is the same except that in this case the two arcs of C cut the rectangle on the same ordinate. One of the two continua into which the rectangle is divided consists of points (x, y) for which x is in the interval common to $[\xi_1, \xi]$ and $[\xi, \xi_2]$, and y is between the correresponding values of $f_1(x)$ and $f_2(x)$. The other points of the rectangle form the other continuum.

If a curve has a finite number of exceptional points of the kind described above, but elsewhere satisfies the condition of § 1, *it divides the plane into two continua, an interior and an exterior.*

PRINCETON UNIVERSITY,
December, 1905.

DETERMINATION OF ASSOCIATED SURFACES.

BY DR. BURKE SMITH.

(Read before the American Mathematical Society, October 28, 1905.)

IT is here proposed to develope a set of formulas for the cartesian coordinates of surfaces which are associate to a given surface referred to its asymptotic lines. The coordinates of an associated surface may thus be found directly from those of the given surface, by differentiation and substitution, when one has solved an equation of the Laplace type.

Since the images on the Gauss sphere of the asymptotic lines on any surface S_0 are the same as the images of a conjugate system of lines on the associated surface S, we may find S by determining the surface which has, as images of a conjugate system, the lines on the sphere which are the images of the asymptotic lines on S_0. Suppose S_0 is referred to its asymptotic lines, and let e, f, g represent the fundamental magnitudes of the corresponding sphere. Then in order that the lines $u =$ const., $v =$ const., on the sphere should be the images of the asymptotic lines on S_0 it is necessary and sufficient that

$$\frac{\partial}{\partial u}\left\{\begin{matrix}1 & 2\\ 1\end{matrix}\right\}' = \frac{\partial}{\partial v}\left\{\begin{matrix}1 & 2\\ 2\end{matrix}\right\}', \tag{1}$$

where the symbols of Christoffel are formed for the sphere. To find the coordinates of a surface S which has the above lines as images of a conjugate system, we proceed as follows: If X, Y, Z are the direction cosines of the normals to S along the conjugate lines, the cartesian coordinates x, y, z of S may be obtained by solving the equations *

$$\begin{cases} xX + yY + zZ = W, \\ x\dfrac{\partial X}{\partial u} + y\dfrac{\partial Y}{\partial u} + z\dfrac{\partial Z}{\partial u} = \dfrac{\partial W}{\partial u}, \\ x\dfrac{\partial X}{\partial v} + y\dfrac{\partial Y}{\partial v} + z\dfrac{\partial Z}{\partial v} = \dfrac{\partial W}{\partial v}, \end{cases} \tag{2}$$

where W is a solution, linearly independent of X, Y, Z, of the equation,

* Bianchi-Lukat, Vorlesungen über Differential-Geometrie, p. 139.

$$\frac{\partial^2 W}{\partial u \partial v} - \left\{ \begin{matrix} 1\,2 \\ 1 \end{matrix} \right\}' \frac{\partial W}{\partial u} - \left\{ \begin{matrix} 1\,2 \\ 2 \end{matrix} \right\}' \frac{\partial W}{\partial v} + fW = 0. \tag{3}$$

Now (3) has equal invariants by virtue of (1). If we put $W = \theta \rho^{-\frac{1}{2}}$, where $-\rho^{-2} = K_0$ is the curvature of S_0, and take account of Codazzi's equations, which in this case are

$$\frac{\partial \log \rho}{\partial u} = -2 \left\{ \begin{matrix} 1\,2 \\ 2 \end{matrix} \right\}', \quad \frac{\partial \log \rho}{\partial v} = -2 \left\{ \begin{matrix} 1\,2 \\ 1 \end{matrix} \right\}',$$

we may write (3) in the form

$$\frac{\partial^2 \theta}{\partial u \partial v} \left(\frac{1}{\sqrt{\rho}} \frac{\partial^2 \sqrt{\rho}}{\partial u \partial v} - f \right) \theta. \tag{4}$$

X, Y, Z are particular solutions of (3), and putting $X = \xi/\sqrt{\rho}$, $Y = \eta/\sqrt{\rho}$, $Z = \zeta/\sqrt{\rho}$, ξ, η, ζ will be particular solutions of (4). Solving (2) for x, y, z and remembering that $\sqrt{eg - f^2} = [X, \partial Y/\partial u, \partial Z/\partial v]$, we have,

$$x = \frac{[W, \partial Y/\partial u, \partial Z/\partial v]}{\sqrt{eg - f^2}}, \quad y = \frac{[X, \partial W/\partial u, \partial Z/\partial v]}{\sqrt{eg - f^2}},$$
$$z = \frac{[X, \partial Y/\partial u, \partial W/\partial v]}{\sqrt{eg - f^2}}. \tag{5}$$

Now by the formulas of Lelieuvre, the coördinates x_0, y_0, z_0 of S_0 satisfy six relations of the form

$$\frac{\partial x_0}{\partial u} = \zeta \frac{\partial \eta}{\partial u} - \eta \frac{\partial \zeta}{\partial u}, \quad \frac{\partial x_0}{\partial v} = \eta \frac{\partial \zeta}{\partial v} - \zeta \frac{\partial \eta}{\partial v}, \tag{6}$$

etc., and since $\rho^2 = \xi^2 + \eta^2 + \zeta^2$ we may write in place of (5)

$$\left\{ \begin{aligned} x &= \frac{\theta \dfrac{\partial^2 x_0}{\partial u \partial v} - \dfrac{\partial \theta}{\partial u} \dfrac{\partial x_0}{\partial v} - \dfrac{\partial \theta}{\partial v} \dfrac{\partial x_0}{\partial u}}{\rho^{\frac{1}{2}} \sqrt{eg - f^2}} = \frac{\theta \dfrac{\partial^2 x_0}{\partial u \partial v} - \dfrac{\partial \theta}{\partial u} \dfrac{\partial x_0}{\partial v} - \dfrac{\partial \theta}{\partial v} \dfrac{\partial x_0}{\partial u}}{\xi \dfrac{\partial^2 x_0}{\partial u \partial v} - \dfrac{\partial \xi}{\partial u} \dfrac{\partial x_0}{\partial v} - \dfrac{\partial \xi}{\partial v} \dfrac{\partial x_0}{\partial u}} \\ y &= \frac{\theta \dfrac{\partial^2 y_0}{\partial u \partial v} - \dfrac{\partial \theta}{\partial u} \dfrac{\partial y_0}{\partial v} - \dfrac{\partial \theta}{\partial v} \dfrac{\partial y_0}{\partial u}}{\rho^{\frac{1}{2}} \sqrt{eg - f^2}} = \frac{\theta \dfrac{\partial^2 y_0}{\partial u \partial v} - \dfrac{\partial \theta}{\partial u} \dfrac{\partial y_0}{\partial v} - \dfrac{\partial \theta}{\partial v} \dfrac{\partial y_0}{\partial u}}{\eta \dfrac{\partial^2 y_0}{\partial u \partial v} - \dfrac{\partial \eta}{\partial u} \dfrac{\partial y_0}{\partial v} - \dfrac{\partial \eta}{\partial v} \dfrac{\partial y_0}{\partial u}} \end{aligned} \right. \tag{7}$$

$$\left|\begin{array}{l} \\ z = \dfrac{\theta\dfrac{\partial^2 z_0}{\partial u\partial v} - \dfrac{\partial\theta}{\partial u}\dfrac{\partial z_0}{\partial v} - \dfrac{\partial\theta}{\partial v}\dfrac{\partial z_0}{\partial u}}{\rho^3\sqrt{eg-f^2}} = \dfrac{\theta\dfrac{\partial^2 z_0}{\partial u\partial v} - \dfrac{\partial\theta}{\partial u}\dfrac{\partial z_0}{\partial v} - \dfrac{\partial\theta}{\partial v}\dfrac{\partial z_0}{\partial u}}{\zeta\dfrac{\partial^2 z_0}{\partial u\partial v} - \dfrac{\partial\zeta}{\partial u}\dfrac{\partial z_0}{\partial v} - \dfrac{\partial\zeta}{\partial v}\dfrac{\partial z_0}{\partial u}} \\ \\ \end{array}\right.$$

where θ is a solution of (4) which is linearly independent of ξ, η, ζ. These equations give the coordinates x, y, z of the associated surface S from those of S_0 when θ is known.

As an example, consider the sphere. The asymptotic lines on the sphere are its imaginary generatrices, and when referred to these lines its equations take the form

$$x_0 = a\left(\frac{1+uv}{u+v}\right), \quad iy_0 = a\left(\frac{1-uv}{u+v}\right), \quad z_0 = a\left(\frac{v-u}{u+v}\right).$$

By direct calculation we have,

$$ds^2 = \frac{-4a^2}{(u+v)^2}\,du\,dv, \quad K = \frac{1}{a^2}, \quad \rho = ia.$$

Equation (4) becomes in this case, $\partial^2\theta/\partial u\partial v = 2\theta/(u+v)^2$, of which the general solution is

$$\theta = 2(U+V)/(u+v) - U' - V',$$

where U and V are arbitrary functions of u and v respectively and the primes denote derivatives. Substituting in (7) we have, as the equations of the associate of a sphere,

$$\begin{aligned} x &= \tfrac{1}{2}a(U - uU' + \tfrac{1}{2}(u^2-1)U'') + \tfrac{1}{2}a(V - vV' + \tfrac{1}{2}(v^2-1)V'') \\ y &= \tfrac{1}{2}ia(U - uU' + \tfrac{1}{2}(u^2+1)U'') + \tfrac{1}{2}ia(V - vV' + \tfrac{1}{2}(v^2+1)V'') \\ z &= \tfrac{1}{2}a(uU'' - U') - \tfrac{1}{2}a(vV'' - V'). \end{aligned}$$

These are the equations of the minimal surfaces.

Consider also the pseudosphere whose equations in terms of the parameters of the asymptotic lines are

$$(8) \qquad \begin{cases} x_0 = i\,\mathrm{csch}\,(u+v)\cos(u-v), \\ y_0 = i\,\mathrm{csch}\,(u+v)\sin(u-v), \\ z_0 = u+v-\coth(u+v). \end{cases}$$

For the square of the lineal element we have

$$ds^2 = du^2 + 2[\mathrm{csch}^2(u+v) + \coth^2(u+v)]dudv + dv^2$$

and $K = -\rho^{-2} = -1$.

Equation (4) becomes in this case,

$$\partial^2\theta/\partial u\partial v = [\operatorname{csch}^2(u+v) + \coth^2(u+v)]\theta.$$

To every solution θ of this equation, which is linearly independent of ξ, η, ζ, we have from (7)

$$(9)\quad \begin{cases} x = \theta\coth(u+v)\cos(u-v) + \tfrac{1}{2}(\theta_u + \theta_v)\cos(u-v) \\ \qquad\qquad - \tfrac{1}{2}(\theta_u - \theta_v)\tanh(u-v)\sin(u-v), \\ y = \theta\coth(u+v)\sin(u-v) + \tfrac{1}{2}(\theta_u + \theta_v)\sin(u-v) \\ \qquad\qquad + \tfrac{1}{2}(\theta_u - \theta_v)\tanh(u-v)\cos(u-v), \\ z = i\theta + \tfrac{1}{2}i(\theta_u + \theta_v)\cos(u+v), \end{cases}$$

where the subscripts denote partial derivatives. From (8) we have $\left\{\begin{smallmatrix}1&2\\1\end{smallmatrix}\right\} = \left\{\begin{smallmatrix}1&2\\2\end{smallmatrix}\right\} = 0$. But since $\left\{\begin{smallmatrix}1&2\\1\end{smallmatrix}\right\} = -\left\{\begin{smallmatrix}1&2\\1\end{smallmatrix}\right\}'$ and $\left\{\begin{smallmatrix}1&2\\2\end{smallmatrix}\right\} = -\left\{\begin{smallmatrix}1&2\\2\end{smallmatrix}\right\}'$ we have for (9), $\left\{\begin{smallmatrix}1&1\\2\end{smallmatrix}\right\} = 0$ and $\left\{\begin{smallmatrix}2&2\\1\end{smallmatrix}\right\} = 0$. Hence (9) are the equations of surfaces of Voss, since the parametric lines consist of two families of geodesics which form a conjugate system.*

If the general solution θ of any equation of the form $\partial^2\theta/\partial u\partial v = M\theta$ is known, where M is any function of u and v, and one can find three particular solutions ξ, η, ζ linearly independent of θ, the equations of a surface S_0 referred to its asymptotic lines may be found by quadrature from (6). The surfaces which are associated to S_0 may be found by substitution in (7). Thus if one can solve an equation of the above type and can choose the three particular solutions ξ, η, ζ linearly independent of θ, the cartesian coordinates of pairs of associated surfaces may be found by six quadratures and substitution in (7). As an example, the writer has already considered the equation $\partial^2\theta/\partial u\partial v = 0$ elsewhere.†

Consider also the equation

$$\partial^2\theta/\partial u\partial v = 2\theta/(u+v)^2$$

of which the general solution is

$$\theta = 2(U+V)/(u+v) - U' - V'.$$

* See Voss, *Sitzungsberichte der Bayrischen Akademie*, 1888, pp. 95–102.
† BULLETIN, Jan. 1906.

As three particular solutions, take

$$\xi = 2U/(u+v) - U', \quad \eta = 2V_1/(u+v) - V_1', \quad \zeta = 2V_2/(u+v) - V_2'.$$

Substitution in (6) gives for S_0 in this case,

$$x_0 = 2(V_2V_1' - V_1V_2')/(u+v) + \int (V_1'V_2'' - V_2'V_1'')dv,$$
$$y_0 = 2(V_2U' + V_2'U)/(u+v) - U'V_2',$$
$$z_0 = -2(V_1U' + V_1'U)/(u+v) + U'V_1'.$$

Substitution in (7) gives at once the equations of the group of associated surfaces.

The method above outlined for determining associated surfaces may be applied to the problem of determining surfaces which admit of continuous deformation with preservation of conjugate lines, since such surfaces are the associates of those whose curvature in terms of the parameters of the asymptotic lines is of the form $K = [\phi(u) + \chi(v)]^{-2}$.* In that case (4) may be written in the form

$$\frac{\partial^2\theta}{\partial u \partial v} = \left(-\frac{1}{4\rho^2}\frac{\partial\rho}{\partial u}\frac{\partial\rho}{\partial v} - f\right)\theta.$$

PURDUE UNIVERSITY,
December, 1905.

NOTE ON THE PRACTICAL APPLICATION OF STURM'S THEOREM.

BY J. E. WRIGHT, M.A.

IT is strange that the following small point is not mentioned in the text-books. The only remark I can find bearing on it is in Encyklopädie der Mathematik I., 1, 4, page 417. Suppose $f, f_1, \cdots, f_n$ are the successive Sturmian functions in *e. g.*, the case when f has only simple roots. If all the real roots of f_r are known, then we can separate the roots by using $f, f_1, \cdots, f_r$ only. For let a, b be two consecutive roots of f_r. Then between a and b f_r does not vanish, and therefore no alteration in the number of changes of sign can arise from the functions $f_r, \cdots, f_n$.

* Bianchi-Lukat, l. c., p. 337.

Hence the number of roots of f between a and b is determined from $f, \cdots, f_r$ only. Practically to avoid zeros of f_r, we substitute $a + \epsilon$, $b - \epsilon$ in the first $r + 1$ functions, when ϵ is small. To complete the work it is necessary to discover whether or not any of the roots of f_r are roots of f, and to apply the theorem to all the intervals given by $\pm \infty$ and the roots of f_r.

Example:

$$f(x) = x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1,$$

$$f_1(x) = 5x^4 - 20x^3 + 27x^2 - 18x + 5, \quad f_2(x) = x^3 - x.$$

Roots of f_2 are $-1, 0, +1$; of these $+1$ is a root of $f(x)$.

	$-\infty$	$-1-\varepsilon$	$-1+\varepsilon$	$-\varepsilon$	$+\varepsilon$	$+1-\varepsilon$	$+1+\varepsilon$	$+\infty$	
f	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$+$	Altogether three real roots.
f_1	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$+$	
f_2	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	
	no roots		no roots		one root		one root		

BRYN MAWR,
January, 1906.

THE MOVEMENT FOR REFORM IN THE TEACHING OF MATHEMATICS IN PRUSSIA.

Ueber eine zeitgemässe Umgestaltung des mathematischen Unterrichts an den höheren Schulen. F. KLEIN. Leipzig, 1904, pp. 82.

Beiträge zur Frage des Unterrichts in Physik und Astronomie an den höheren Schulen. Vorträge gehalten von Behrendsen, Bose, Riecke, Stark, und Schwarzschild. E. RIECKE. Leipzig, 1904, pp. 108.

Neue Beiträge zur Frage des mathematischen und physikalischen Unterrichts an den höheren Schulen. F. KLEIN und E. RIECKE. Teil I. Leipzig, 1904, pp. 190.

Verhandlungen der Breslauer Naturforscher-Versammlung über den naturwissenschaftlichen und mathematischen Unterricht an den höheren Schulen. Leipzig, 1905, pp. 77.

Bericht der Unterrichts-Kommission der Gesellschaft deutscher Naturforscher und Aerzte über ihre bisherige Tätigkeit. Leipzig, 1905, pp. 57.

THESE works are recent manifestations of an important movement for the improvement of the teaching of mathematics

in Prussia, which has been in active progress for quite a number of years.

At the head of the movement stands Professor Klein, of Göttingen, whose views as to the reform needed are expressed in the first work of the above list. This contains five lectures delivered by Klein at a vacation course for teachers held at the University of Göttingen in the spring of 1904. In addition to these there are reprinted several previous publications of Klein's, and a paper by E. Götting on the aim of mathematical instruction in the higher schools (15 pages). The second work contains lectures on topics relative to physics and astronomy delivered at the same course for teachers, and the third consists of the first two bound in one volume.

At the outset, Klein defends himself against two accusations that are usually made against university professors who take active interest in the problems of the schools: first, that they wish to increase the requirements in mathematics; second, that they have in mind exclusively preparation for university work in mathematics. Klein denies emphatically that he takes either of these attitudes. He wishes no raising of the mathematical level in the schools, but rather its horizontal displacement. He is not concerned about what mathematics the prospective student of mathematics in the university may learn in the schools; this student will be taken care of when he gets to the university. It is the non-mathematician that needs special attention — the prospective physician, chemist, jurist. The primary duty of the schools is to give these a mathematical equipment that shall be really fruitful in their subsequent careers.

Klein advocates, not an enlargement of the content of elementary mathematics, but a horizontal displacement of its boundary. Purists define elementary mathematics as the totality of those parts of mathematics in which the concept of limit is not used. This admits much of the theory of numbers, for example, into elementary mathematics, but excludes $\sqrt{2}$ and π. A second definition admits the idea of limit but not in the special form denoted by the symbols dy/dx and $\int ydx$. This would exclude from elementary mathematics the very common ideas of tangent to an arbitrary curve, of its length and area, of velocity, acceleration, etc., and would admit much that is not within the range of the school, for example, an "elemen-

tary theory of analytic functions of a complex variable," which, with Weierstrass, avoids Cauchy's theorem and bases all proofs on comparison of infinite power series. A third definition calls everything elementary in geometry which relates to the geometry of Euclid and the ancients. This would admit into elementary geometry the difficult modern researches on the foundations of geometry, and would exclude the simplest modern considerations on variability and transformation of figures; that is, it would keep the spirit of modern geometry entirely out of the schools.

The only definition of elementary mathematics that will be of any use to the school must be a practical one:

In all domains of mathematics those parts are to be called elementary which can be understood by a pupil of average ability without long continued special study.

The content of elementary mathematics, under this definition, is naturally dependent on the time. Improved presentation brings new topics into the elementary field. There can be no question that the determination of the derivative and the integral of x^n (n a positive integer) to-day belongs in the elementary field, and Klein's chief thesis is:

The introductory development of the function concept, the first introduction to analytic geometry, the beginning of the differential and integral calculus, should be a part of the course of every pupil.

The explanation and discussion of this thesis constitutes the backbone of the series of addresses. The early and continued use of graphic representation of simple functions is urged; for example of

$$y = ax + b, \quad y = x^2, \quad y = 1/x$$

in "Untersekunda" (age 14–15), together with varied applications, especially to the mathematical discussion of physical phenomena. As a minimum, the law of falling bodies, the motion of the pendulum and wave motion should be understood mathematically. The treatment must be sufficiently formal to assure real understanding, but the main end is clear grasp of the fundamental concepts and their concrete significance.

"It is often said that the beginning of higher mathematics (*i. e.*, of the differential and integral calculus) means a revolution in the thinking of the student. The plan of instruction which I am urging is meant to spare the student this revolu-

tion by accustoming him from the beginning to the processes of thought which are later to prove fruitful. This 'revolution' is merely a test of the unsuitability of the earlier work to the later aims."

It would be interesting to cite many other instructive details of these addresses. In fact the whole work is well worth translating into English, but what has been said above must suffice for the present writing, with the remark in conclusion that here, as elsewhere, Klein also takes occasion to lay stress on the demand that the universities give special attention to the needs of prospective teachers of mathematics. It will be recalled that three years' university study of mathematics are a part of the professional preparation of every teacher of secondary mathematics in Prussia. Consequently, it is likely that prospective teachers constitute the majority of the mathematical students at the universities, and yet until recently the latter have given no explicit attention to their needs. The subject matter and the methodology of elementary mathematics deserve special attention from the university, and Klein announces for the winter next following a course by himself for prospective teachers under the title: "Concerning mathematical instruction in the higher schools."

The fourth and fifth reports relate to activities of the German society of natural scientists and physicians, dating from the year 1901, when the so-called "Hamburg theses" relative to instruction in the biological sciences were presented. In the consideration of these theses at the session at Cassel in 1903, it was decided, upon motion of Professor Klein, to make the entire field of instruction in the natural sciences and mathematics the subject of thorough discussion. This discussion took place at Breslau in 1904 and No. 4 of the above list is the published report of this discussion. Among the formal papers presented were one by Fricke on "The present status of instruction in the natural sciences and mathematics in the higher schools" and one by Klein under the title "Remarks on instruction in mathematics and physics." Here also Klein urged the importance of the function concept and of the elements of the calculus in the mathematical side of modern culture, the need of considering mathematics in closer relation with its applications in the exact sciences and in all phases of life capable of precise formulation, and the danger of allowing the formal

logical side to predominate too strongly in instruction in mathematics. Both speakers lay marked stress on the importance of thorough preparation of teachers, and emphasize the duty of the universities in this connection, not only in the matter of scientific equipment, but also in that of methodic and didactic preparation. Klein also advocates that teachers be given regularly a semester's leave of absence to give them opportunity to survey the progress of their science, and by personal relations and inspection on travels here and there, to make acquaintance with the progress that is being made in this field, and especially to see what part their own subject is playing in the culture of our day.

At the close of this discussion, a commission of twelve was appointed, including Klein and other mathematicians, to draw up courses of study in mathematics and the natural sciences, bringing into unified and practical form the various propositions made in the discussion. The report of this commission is the fifth work named above, and an educational document of great significance.

After a general outline by the chairman, Professor A. Gutzmer, of the University of Jena, of the work of the commission, its leading recommendations and the principles which governed it in making them, there follow detailed curricula in each of the subjects mathematics, physics, chemistry, botany, and zoology. Only that in mathematics concerns us here. Not undervaluing logical training, the commission regarded strengthening of space intuition and training to the habit of functional thinking as the most important task of instruction in mathematics. As the goal to be reached by this instruction there were set up a scientific survey of the organization of the whole field covered in the school, a certain facility of regarding things mathematically, and above all insight into the importance of mathematics in the exact knowledge of nature, and in the culture of the day in general.

In accordance with this general point of view, the detailed curriculum gives new prominence to the concrete phases of mathematics (drawing, graphic methods), to the function concept and the representation of functions by curves, and to the applications of mathematics. The introduction of the elements of the calculus was left to the option of the teacher.

Though relating directly only to German conditions, these publications are of significance beyond the borders of that country. The first and the last will prove of most interest to

the general reader. The movement itself, of which all these works are the outgrowths, is undoubtedly of the first importance. In addition to active interest on the part of leading mathematicians and of the strong and influential society appointing and supporting the commission whose report we have considered, other evidences of a widespread and significant participation in the movement are not lacking. Of these only the representation of the Deutsche Mathematiker-Vereinigung on the Commission, and the space devoted to pedagogic matters in its *Jahresbericht*, need be mentioned here. While this movement is directly concerned only with mathematics in Germany, it is nevertheless of international significance, both on account of the fundamental nature of the changes which it proposes and the weight attaching to the names of the societies and men that are engaged in it. In its essentials, it is in close harmony with movements of the day for improvement of the teaching of mathematics in England, in France and in the United States, which agree in demanding:

1. That the presentation take more careful account of the pupils' powers; that it be psychological as well as logical; that consequently the intuitional, the inductive, the concrete, be given prominent place in the teaching of mathematics.

2. That the amount of complex or merely technical work be diminished.

3. That, in addition to its disciplinary value, the importance of mathematics for a clear understanding of nature, and the fundamental character of mathematics in the fabric of modern civilization be brought effectively before the pupils.

J. W. A. YOUNG.

THE UNIVERSITY OF CHICAGO,
January 24, 1906.

VECTOR ANALYSIS.

Vorlesungen über die Vektorenrechnung. Von E. JAHNKE. Leipzig, B. G. Teubner, 1905. xii + 235 pp.

THE entering class at an American college is more or less prepared on elementary geometry and algebra, and perhaps on trigonometry. The problem of collegiate instruction has, therefore, one fairly well-defined premise. All the students who continue their work in mathematics, for no matter what purpose, must study analytic geometry and calculus. From this

point on, however, the needs of different students are different, and to a considerable extent the aims and the subjects of instruction must vary. What shall be the place, and what the meaning of further instruction in algebra?

It is possible to proceed in several ways. In the first place the plan usually followed may be adopted and more algebra, similar to that with which the student is already familiar, may be taught. On the other hand the questions of the foundations of algebra and arithmetic may be studied with the object of showing the student the meaning of postulates, the significance of the number system, and the fact that algebra is merely symbolic and devoid of any inherent concrete interpretation. The present fervor for logic may make this course of procedure popular and in the future much time may be devoted to this branch of algebra. The result, however, might turn out to be far from justifying the expenditure in time and exertion. It is difficult to get any real grasp of the meaning of postulates or any considerable enjoyment in the logical development of a subject from them, unless the consequences of other related systems of postulates be also investigated. Thus it would seem better to postpone this second extension of algebra until such time as similar questions in geometry with especial reference to Lobatchevskian and Riemannian geometry shall be taken up. In the third place it may be the aim of the instructor to broaden the student's ideas on the general subject of algebra, to bring out the natural generalization of those ideas, to show how geometry and algebra may be united into a geometric algebra somewhat in contrast to algebraic or analytic geometry.

It seems as if this third way of continuing algebra, no more difficult of execution and considerably more practical than the preceding two, is all too much neglected here in America and in the world at large. The method of advance almost universally followed in respect to algebra is as if on entering college the student were put to work studying the nature of postulates and the formal side of geometry or developing further the synthetic methods of higher geometry instead of proceeding to analytic geometry and through it to the generalizations of modern geometry. If a thorough course (or even two) in analytics — an amount considerably in excess of that required for elementary calculus and its applications — be presented before matters of postulates and higher synthetic methods, why should not some corresponding training in geometric algebra

come before the investigation of those other algebraic questions? The reason is perhaps more historical than rational. And although it would doubtless be gross exaggeration to state that geometric algebra is on a par with analytic geometry as regards its general importance, yet in view of its great usefulness in geometry and its present widespread use in physics, not to mention its own intrinsic interest, it certainly deserves a greater relative recognition than is now given to it.

To further this end nothing seems more propitious than the appearance of Jahnke's Lectures on vector analysis. The title is but poorly descriptive of the book, which is far wider in its scope and method. These lectures are really lectures on multiple algebra and form an excellent introduction to the subject — no more than an introduction because the theory of the linear function which is of such fundamental importance is hardly mentioned and the closely allied theory of linear associative algebra is not touched at all. The author's method of treatment is logical and pedagogic from beginning to end. He first considers in detail the algebra of the plane and only later proceeds to corresponding discussions for space. The similarities between the two cases render the study of space much easier after the plane has been treated; and the dissimilarities bring out very clearly the changes that must still be made in order to generalize to space of n dimensions. What is especially remarkable for a German text-book is that not merely does the author work out numerous examples but he actually leaves a larger number as exercises to be worked out by the student. This feature of the book will certainly appeal to any one who may wish to teach this subject, in which collections of problems are hard to find.

In the first chapter the beginnings of Grassmann's point analysis, the addition and subtraction of points affected with numerical coefficients, is taken up. This leads at once to the free vector as the difference of two points and to the addition and subtraction of vectors. The third chapter discusses localized vectors and the multiplication of points. Next comes the multiplication of free vectors followed by a large and various set of applications to problems in mechanics and physics. A sixth chapter on regressive multiplication completes the treatment of the plane. From this brief indication of the contents of the first eighty-eight pages of the Lectures, it can be seen that the reader has been brought step by step to the different and mani-

fold ideas and methods underlying geometric algebras of the non-linear-associative type. The study of the complex variable will furnish a correspondingly simple case of this latter type.

Proceeding to space, the author follows exactly the same program, weaving together side by side the point analysis and the vector analysis. Here, however, there are two classes of directed quantities (vectors and bivectors or lines and planes) and the product of three vectors or four points is a scalar; whereas in the plane there was only a single kind of vector and the product of two vectors was a scalar. Among the applications may be mentioned the theory of moments of inertia, of the screw with Chasles's theorems, and of the null-system. Further there is a treatment of differentiation, about thirty pages in the whole, less than ten of which are on divergence, curl, and Gauss's theorem. Stokes's theorem seems not to be mentioned. This insignificant amount of attention paid to space differentiation and integration, which makes the book much less useful than many another to the physicist, leaves greater room for the purely algebraic and geometric side and is, therefore, particularly to be commended from the standpoint of one wishing to learn or to teach multiple algebra as a continuation of elementary algebra and of a little analytic geometry.

There is one point on which a little adverse criticism or at any rate a word of caution seems necessary. On page 110 the author says: "In most books on vector analysis it is a question of vector and scalar products, not of outer and inner products. While the inner and scalar products are identical, there is an essential difference between the outer and vector products. If [**ab**] be an outer product, [**ab**] is a vector of higher dimensions than **a** or **b** individually. On the other hand $V\mathbf{ab}$ or $\mathbf{a} \times \mathbf{b}$ is a simple vector **c**. In this kind of multiplication the conception of dimensions is lost." In a paper read at the congress at Heidelberg, I have analysed this argument in detail and come to the conclusion that if it be not wrong it is, at least from the point of view of the physicist, so inadequate as to be worthless.* There is no need of repeating the analysis here. The trouble arises through the use of the term dimensions in two different senses. There is just one fundamental proposition underlying the idea of physical dimensions, namely, that the physical dimensions of a product are the product of the dimensions of the individual factors. This is independent of the

* "On products in additive fields," Verhandlungen des III. internationalen Mathematiker-Kongresses in Heidelberg, 1904, § 5, pp. 209–211.

analysis by which such product is represented. It is even independent of the kind of product. Thus if **a** and **b** are vectors representing length, the scalar product $\mathbf{a} \cdot \mathbf{b}$, the inner product $[\mathbf{a}|\mathbf{b}]$, the vector $\mathbf{a} \times \mathbf{b}$, the bivector $[\mathbf{ab}]$, the dyad **ab**, the quaternion **ab**, all have the dimensions $[L]^2$. It will not do for the physicist to lose sight of the fact for a moment. The followers of Grassmann, however, use the term dimension in a more geometric sense. If **a**, **b**, **c** be vectors representing lengths, **ab** is a bivector or vector of the second class (zweiter Stufe) and **abc** is a trivector, a scalar, a quantity of the third class. Again, by the law of regressive multiplication, (**abc**)**a** becomes a vector of the first class. As a matter of fact the physical dimensions of (**abc**)**a** are $[L]^4$. As a geometer one may take great delight in the theory of geometric dimensions in multiple algebra, but as a physicist he must discard it as confusing. This is precisely what is done by the physicists who use Grassmann's notation.

The whole matter may, however, be let pass with good grace inasmuch as the book is otherwise singularly free from blemishes and well fulfils the requirements for an elementary text on geometric or multiple algebras. Jahnke's lectures supplemented first with the elementary geometric theory of imaginary numbers and of quaternions and second with Gibbs's theory of dyadics, where the ideas of products are considerably more extended than anywhere else, would form an excellent introductory course on higher algebra and would furnish, in addition, a very respectable knowledge of some parts of projective geometry, mechanics, and physics.

EDWIN BIDWELL WILSON.

YALE UNIVERSITY,
January, 1906.

CELESTIAL MECHANICS.

An Introduction to Celestial Mechanics. By F. R. MOULTON, Ph.D., Instructor in Astronomy in the University of Chicago. New York, The Macmillan Company, 1902. xv + 384 pp.

THE need of a work written in the English language which would properly introduce astronomical students into the subject of celestial mechanics has been keenly felt for some time. In many of our universities the teaching of astronomy is confined to practical and to what is commonly called theoretical

astronomy, the latter involving chiefly the derivation of suitable formulas for the numerical solution of parabolic, elliptic, and hyperbolic orbits and of special perturbations. As a result of this, the young astronomer often enters upon his work as an observer without a proper understanding of the general problems of astronomy to the solution of which his observations are to contribute, or as a mechanical computer without a real insight into the methods which he is applying, or both. It is probably not far from true that the sum total of the knowledge in celestial mechanics of many American students at the close of their university career is confined to the subject matter of Watson's Theoretical astronomy, supplemented sometimes only by a mechanical working familiarity with the excellent collections of formulas in von Oppolzer's Lehrbuch zur Bahnbestimmung. The importance of this subject is considered so great that two works along these lines have appeared in Germany in rather close succession: a new edition by Dr. Buchholz, Göttingen, of Klinkerfuss's Theoretische Astronomie, and Bauschinger's Bahnbestimmung, just off the press. All of these serve an excellent, though very special purpose, and are priceless in the hands of the teacher, but if properly absorbed, they perhaps take more than their just proportion of the student's time. They are, unfortunately, not directly designed to develop a taste nor the necessary preparation for the study of the methods of investigation and the results attained in celestial mechanics, such as would inspire to the study of Tisserand's Mécanique céleste, or Poincaré's Les nouvelle méthodes de la mécanique céleste, or Brown's Lunar theory, or other standard works along the same lines.

In the opinion of the writer, Dr. Moulton has succeeded in filling this gap and has exhibited rare skill in the selection and treatment of the necessary material. But, to quote from the preface, the work will also be welcomed by mathematical students who "desire to get an idea of the processes by means of which astronomers interpret and predict celestial phenomena." Moulton's work will develop in the astronomical student a desire for the study of the advanced mathematics necessary for further pursuit of celestial mechanics, and in the mathematical student a greater love for his subject through the practical applications to which he sees it put. It will serve, no doubt, to draw the astronomer and mathematician into closer relation. The book contains for its size an abundance of material, including the

results of some of the author's own investigations; the treatment is original, clear and concise. Mention should be made in this connection of the method adopted in Chapter VII in developing Lagrange's particular solutions of the problem of three bodies; of his investigations regarding the validity of the expressions for the elements α_i and β_i in power series in m_1 and m_2 (pages 266 and 257); of his general method of determining the elements of an orbit of a comet or planet after the solution of the heliocentric and geocentric distances is accomplished (article 236), etc.

A feature of the work to be particularly commended on account of the breadth which it gives to the subjects treated is the inclusion of applications to astrophysical problems, and of principles useful in geodesy. The importance of the study of celestial mechanics is thus brought home to the student specializing in astrophysics or geodesy. There are articles on the velocity from infinity applied to the escape of atmospheres (Chapter II, Article 36); the heat of the sun, the temperature of meteors, the meteoric theory of the sun's heat, Helmholtz's contraction theory (Chapter II, articles 39–43); Gegenschein (Chapter VII, Article 124).

At the end of each of the eleven chapters is given a carefully selected set of problems, the solution of which will add materially to a clear understanding of the text, and an historical sketch and bibliography which goes to prove a thorough familiarity with the literature of the subject, although in places omissions may be noted of references of at least equal importance to those given. The work is strictly up to date, touching as it does on the most recent researches of Hill, Poincaré, Darwin and others.

In the last two chapters of the book the author attempts to derive the working formulas necessary for the computation of a parabolic or general orbit of a comet or planet. This attempt is a radical departure from the otherwise excellent plan of the book. Although a mere outline of these methods might have been in place, the last two chapters could have been entirely omitted without in the least impairing the excellency of the work. Why should not Hansen's method of general perturbations be given a chapter, if it is considered worth while to consider in detail such very special topics as the adaptation of the solution of the problem of two bodies to the numerical computation of an orbit? In the first place, in the brief space which

the author has devoted to the subject, it is not possible to do this subject justice so as to be of actual practical use to the student. The inclusion also of formulas for the reduction from apparent to mean place, etc., without proof, encourages mechanical computing. Such matters really have no place in an introduction to celestial mechanics. The presentation is patterned somewhat after Watson's Theoretical astronomy, excellent certainly in its time but now improved upon by other works. Take, for instance, the discussion on page 344 of the various fundamental formulas: why not follow Oppolzer's plan of merely stating that six conditions being available for the determination of five elements, one observation is made incomplete by substituting for it the condition that the middle place shall lie in an arbitrary auxiliary circle passed through it; that the formulas become simplest when the great circle is passed through the solar place at the middle date, and that in the so-called "Ausnahmefall" the most accurate result is obtained by adopting an auxiliary great circle through the middle place perpendicular to the geocentric motion. This latter is Oppolzer's method, and does not fail for parabolic orbits. Similarly, Oppolzer's method for determining elliptic orbits is certainly to be preferred to the original method of Gauss owing to its higher degree of approximation. But I consider these matters of really too special a nature to have a place in Moulton's work. As said before, they do not give the astronomical student a real insight into the nature of the numerical problem, nor can they be of great interest to the mathematical student.

Chapter I is designed to leave no doubt in the student's mind regarding the foundations of the subject. There might be a doubt as to whether or not the reader ought to be familiar with most of the elementary principles considered, but a review of them on the basis of Moulton's logical presentation can do no harm.

Chapter II deals with rectilinear motion and applications referred to above. It includes the consideration of forces varying directly as the distance, inversely as the square of the distance, proportional to the velocity, and proportional to the square of the velocity, the general method of solution being given the place it deserves.

Chapter III is on central forces. After a consideration of simultaneous differential equations, the orbits corresponding to a given law of force are discussed, and vice versa. The proper foundation is then laid for a consideration of the universality of

Newton's law on the basis of observed double star orbits, which leads to the conclusion that if the attraction does not depend upon the direction of the bodies from each other, Newton's law of gravitation operates in binary systems.

Chapter IV contains a judicious selection of topics relating to the potential attraction of bodies, and should particularly appeal to the geodesist. Chapter V is on the problem of two bodies. Chapter VI gives the fundamental principles of the problem of n bodies.

Chapter VII is on the problem of three bodies. Its subject matter is particularly well chosen, leaving the reader with a clear conception of all that has been accomplished in attempting its solution. The particular solutions are clearly presented, the treatment of the surfaces of zero velocities, the stability of particular solutions, etc., is concise and to the point.

Chapter VIII contains the geometric interpretation of perturbations and is valuable as a means of assisting the student to a proper understanding of the effects of perturbation on the elements of an orbit.

Chapter IX treats of analytic methods of determining perturbations. The object of the chapter is stated by the author in the following words: "The chief difficulties which arise in getting an understanding of the theories of perturbation come from the large number of variables which it is necessary to use, and the very long transformations which must be made in order to put the equations in a form suitable for actual computations. It is not possible, because of the lack of space, to develop in detail the explicit expressions adapted to computation; and indeed it is not desired to emphasize this part, for it is much more important to get an accurate understanding of the nature of the problem, the mathematical features of the methods employed, the limitations which are necessary, the exact places where approximations are introduced, if at all, and their character, the origin of the various sorts of terms, and the foundations upon which the celebrated theorems regarding the stability of the solar system rest" (page 256). The author has performed his task in an exceptionally satisfactory manner.

Chapters X and XI contain the derivation of formulas for the computation of the orbits of comets and asteroids, and have been referred to above.

A. O. LEUSCHNER.

UNIVERSITY OF CALIFORNIA,
BERKELEY ASTRONOMICAL DEPARTMENT,
February 10, 1906.

SHORTER NOTICES.

Géométrie, Premier et Second Cycle. Par EMILE BOREL. Paris, Librairie Armand Colin, 1905. 383 pp.

THIS is one of a series of elementary text-books written by Borel to meet the demand caused by the recent activity for the improvement of instruction in elementary mathematics in France. As the author states, it has been his aim to make a book more in line with the ideas of the geometers and analysts of the nineteenth century who have taught that "Geometry is the study of groups of movements." With this end in view he has introduced the ideas of symmetry and displacements as often as possible, thereby making many demonstrations simpler and more concrete.

The book is divided into three parts. Part one contains about what is ordinarily called books I and II. The treatment however is quite distinctive. The theorems concerning isosceles triangles and parallel lines are proved by the use of symmetry. The symmetry of regular figures is discussed at some length. For use in constructions, the square is introduced as a third instrument and many constructions are thereby made much simpler. Part two is the extension of the first part to space. The ideas of translation and rotation are used quite freely. Part three treats similitude, areas and volumes. The trigonometric functions of the angle are given and used in the demonstration of many of the theorems which follow. The book closes with a short discussion of conics and a chapter devoted to the approximate determination of plane areas. A good collection of problems is given at the end of each part.

C. L. E. MOORE.

Orthogonale Axonometrie. Ein Lehrbuch zum Selbststudium. Mit 29 Figurtafeln in besonderem Hefte. By Dr. RUDOLF SCHÜSSLER, Associate Professor of Mathematics in the Technical School at Graz. Leipzig, Teubner, 1905. 170 pp.

IN this book the attempt has been made to develop axonometry as an independent method of representation without the use of the horizontal and vertical planes of descriptive geometry and without the assumption of any previous knowledge of conical or parallel perspective. The method of procedure is rather similar to that of Pelz* but made much more concrete and ele-

*See the Vienna *Sitzungsberichte*, vols. 81, 83, 90 and the Prague *Sitzungsberichte*, 1885, 1895.

mentary, as well as more comprehensive and systematic. The book begins very concretely; after defining what axonometry is, the projection of the rectangular coordinates of a point, the projections of a line on each of the coordinate planes, and the intersections of a plane with each of the coordinate planes are explained with excellent clearness. The discussions regarding a finite but inaccessible point of intersection of two lines, and of a line and plane (pages 19, 20) are particularly commendable. Each particular or exceptional case is discussed in sufficient detail so that every case met in subsequent constructions is a direct application of the principles already explained.

The treatment of pyramids and prisms is now easy and natural; a thorough treatment of shades and shadows is added, applying to all bodies bounded by planes. The section applied to metrical properties, lengths, normals, angles, etc., is less direct and natural than the preceding ones and the question may well be asked whether for such purposes this method of representation has any advantage over the ordinary h, v method.

The circle is treated with care and the usual detail; a large number of alternate proofs are given for the same theorem, perhaps too many for a student who is reading it alone — and for such students the book was designed — though they are valuable to one already familiar with the projection for purposes of comparison. Now follow conics, cones, cylinders and their curves of intersection, and the ideas here developed are applied to surfaces of revolution. The only slip noticed is on page 153, where it is stated that an inflexion in the meridian curve will produce a corner in the shadow cone. The shadow cone must touch the parallel circle generated by the point of inflexion.

Numerous exercises for the student to solve are added to each section, many of them provided with a figure for suggestion. Notice is always given on the margin when a new figure is considered. Finally, mention should be made of the figures themselves, which were drawn by the author. They are very well done; proper lines are drawn heavier or lighter to show at a glance just what the figure means. The whole work shows care and thoughtfulness on every page, and is provided with an excellent index. It is an important addition to the literature on constructive drawing.

VIRGIL SNYDER.

NOTES.

The Thirteenth Summer Meeting and Fifth Colloquium of the AMERICAN MATHEMATICAL SOCIETY will be held at Yale University during the entire week, September 3–8, 1906. The first two days will be devoted to the regular sessions for the presentation of papers. The Colloquium, which will open on Wednesday morning, will include the following courses of lectures: Professor E. H. MOORE, "On the theory of bilinear functional operators"; Professor MAX MASON, "Selected topics in the theory of boundary value problems of differential equations"; Professor E. J. WILCZYNSKI, "Projective differential geometry."

THE nineteenth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will be held at the Northwestern University, Evanston, Ill., on Saturday, April 14, 1906, the first session opening at 10 A. M.

AT the meeting of the London mathematical society held on February 8, the following papers were read: By Major P. A. MACMAHON, "Partitions of numbers in space of two dimensions"; by DR. H. F. BAKER, "The Eisenstein-Sylvester extension of Fermat's theorem."

THE National academy of sciences will hold its annual meeting at Washington, D. C., April 16–18.

THE American association for the advancement of science will hold a summer session at Ithaca, New York, during the week June 26 to July 2. Dr. EDWARD KASNER is vice-president and Professor L. G. WELD secretary of Section A.

THE foreign universities named below offer advanced courses in mathematics during the summer semester of 1906 as follows:

UNIVERSITY OF HALLE.—By Professor G. CANTOR: Theory of numbers, four hours; Seminar, two hours. — By Professor A. WANGERIN: Differential geometry, five hours; Definite integrals and differential equations, four hours; Selected chapters of potential theory, one hour; Seminar, two hours. — By Professor A. GUTZMER: Differential calculus with exercises, five hours; Theory of functions, four hours; Selected chapters of analytic mechanics, one hour; Seminar, two hours. — By Professor V. EBERHARD: Algebra, I, four hours; Analytic

geometry of conics, two hours. — By Professor F. BERNSTEIN: Historical review of the principal fields of pure mathematics, two hours; Mathematics of insurance, two hours.

UNIVERSITY OF JENA. — By Professor R. HAUSSNER: Differential and integral calculus with exercises, five hours; Plane analytic geometry, four hours; Division and quadrature of the circle, two hours; Seminar, one hour. — By Professor J. THOMAE: Elliptic functions, four hours; Mathematical geography, four hours. — By Professor G. FREGE: Analytic mechanics II., four hours.

UNIVERSITY OF LEIPZIG. — By Professor C. NEUMANN: Applications of the calculus, four hours; Seminar, one hour. — By Profsssor A. MAYER: Analytic dynamics, five hours; with exercises, one hour. — By Professor O. HÖLDER: Applications of elliptic functions, three hours; Selected chapters of elliptic modular functions, two hours; Seminar, one hour. — By Professor K. ROHN: Curves of order three and four, four hours; Determinants, two hours; Seminar, one hour. — By Professor F. HAUSDORFF: Ordinary differential equations, four hours; with exercises, one hour. — By Professor H. LIEBMANN: Plane analytic geometry, four hours; Introduction to algebraic analysis, four hours; with exercises, one hour.

UNIVERSITY OF MUNICH. — By Professor F. LINDEMANN: Integral calculus, five hours; Substitutions and higher algebraic equations, four hours; Mechanics of non-rigid bodies, two hours; Seminar in mechanics, one and one half hours. — By Professor A. VOSS: Introduction to the theory of differential equations, four hours; Analytic geometry of space, five hours; Seminar, two hours. — By Professor A. PRINGSHEIM: Definite integrals, four hours; Applications of elliptic functions, two hours. — By Professor K. DOEHLEMANN: Descriptive geometry (axonometry, perspective), three hours; with exercises, two hours; Synthetic geometry, II, four hours; Imaginary elements in geometry, one hour. — By Professor E. v. WEBER: Determinants, four hours; Differential calculus, four hours; with exercises, two hours. — By Professor H. BRUNN: Elements of higher mathematics, four hours. — By Dr. F. HARTOGS: Selected chapters of the theory of functions, four hours.

UNIVERSITY OF ROSTOCK. — By Professor O. STAUDE: Analytic geometry of space, four hours; Analytic mechanics, four hours; Seminar, two hours.

UNIVERSITY OF WÜRZBURG. — By Professor F. PRYM: Integral calculus, six hours; Proseminar, two hours; Seminar, two hours. — By Professor E. SELLING: Analytic mechanics, four hours; Spherical astronomy, two hours. — By Professor G. ROST: Analytic and synthetic geometry of conics, four hours; Applications of the calculus to plane curves, four hours; Twisted curves and surfaces, four hours; Noneuclidean geometry, two hours; Seminar in pure mathematics, two hours; Seminar in mechanics, two hours.

UNIVERSITY OF BERN. — By Professor J. H. GRAF: Spherical harmonics, three hours; Bessel functions, three hours; Definite integrals, three hours; Differential and integral calculus, two hours; Theory of investments, two hours; Seminar, two hours. — By Professor E. OTT: Differential calculus, two hours; Analytic mechanics, I, two hours. — By Professor G. HUBER: Elliptic integrals with applications, two hours. — By Dr. C. MOSER: Insurance, two hours. — By Dr. J. V. PEXIDER: Theory of numbers, three hours; Elements of the theory of aggregates, one hour; Analytic theory of numbers, two hours; Prime numbers, one hour. — By Dr. L. CRELIER: Synthetic geometry of space, two hours; Geometry of the triangle, with exercises, four hours.

UNIVERSITY OF ZÜRICH. — By Professor H. BURKHARDT: Algebraic analysis, four hours; Mathematical theory of dissipative phenomena, four hours; Seminar, two hours. — By Professor A. WEILER: Analytic geometry, II, four hours; Descriptive geometry, II, four hours; Synthetic geometry, two hours. — By Dr. E. GABLER: Principal theorems of differential and integral calculus, two hours; Political arithmetic, two hours; Content and method of instruction in geometry in preparatory schools, one hour.

DURING the fortnight beginning April 19 a vacation course for gymnasium teachers will be held at the University of Göttingen. Professors Klein and Behrendsen will explain in detail the schedules for mathematics and physics proposed by the commission of the society of German scientists and physicians (see the review of these reports by Professor J. W. A.

Young in the BULLETIN, pages 347–352). Regular courses of lectures will be given by Professors Behrendsen, Prandtl, Runge, Simon, Voigt and Wagner.

BEGINNING on Friday, March 23, Professor H. A. LORENTZ, of the University of Leiden, will give at Columbia University a course of eight lectures on "The theory of electrons and its applications to the phenomena of light and radiant heat." These lectures will be given on Fridays from 4 to 6 p. m. and on Saturdays from 10 to 12 a. m., March 23 to April 7, and from 4 to 6 p. m. on Wednesday, April 11, and Thursday, April 12.

THREE senior wranglers in mathematics sit in the British court of appeal — Fletcher Moulton, just elevated to the bench, Lord Justice Ronier and Lord Justice Sterling. Mathematicians are also strongly represented at present among the high court judges; the Lord Chief Justice Buckley and Justices Channell, Joyce and Bray were all wranglers, and Justice Sutton was a senior optime.

THE Smith's prizes for the present year have been awarded to C. F. RUSSELL, of Pembroke College, for his essay "The geometric interpretation of apolar binary forms," and to F. J. M. STRATTON, of Gonville and Caius college, for his essay "A problem in tidal evolution suggested by the motion of Saturn's ninth satellite."

THE exposition of arts and industries held in Brussels in the fall of 1905 awarded its gold medal to the periodical *l'Enseignement mathématique* for its services to mathematical pedagogy.

PLANS are being perfected for preparing a suitable memorial and monument to the late Professor GUIDO HAUCK in the rooms of the technical school at Berlin, at which he was professor for twenty-seven years and thrice rector. A general invitation for contributions has been issued, signed by representatives of most of the academies, universities and technical schools of Germany. Remittances may be sent to Herr Kiesel, Sekretär der Technischen Hochschule, Berlin-Charlottenburg.

THE mathematical society of Berlin announces that the hindrances which prevented the erection of a suitable monument over the grave of WEIERSTRASS no longer exist and that the

stone will be placed in position this spring. Friends and students of Weierstrass who wish to contribute for this purpose may send their contributions to Professor Dr. J. Knoblauch, Berlin W, Karlsbad 12.

DR. E. ZERMELO, of the University of Göttingen, has been promoted to an assistant professorship of mathematics.

DR. F. HARTOGS has been appointed docent in mathematics at the University of Munich.

PROFESSOR G. HETTNER, of the technical school at Berlin, has been awarded the order of the crown of the third class by the German emperor.

DR. L. BOULANGER has been appointed professor of mechanics at the University of Lille.

DR. M. F. W. DYSON, of the observatory at Greenwich, has been appointed professor of astronomy at the University of Edinburgh.

DR. R. FUCHS has been appointed docent in mathematics, and Dr. REISSNER has been promoted to an associate professorship of mechanics at the technical school of Berlin.

MR. E. T. WHITTAKER, of Cambridge, has been appointed Andrews professor of astronomy at the University of Dublin, and astronomer royal of Ireland.

PROFESSORS T. J. I'A. BROMWICH, of Queen's College, Galway, and J. H. JEANS, of Princeton University, have been elected fellows of the Royal society of London.

PROFESSOR W. SCHEIBNER, of the University of Leipzig, celebrated his eightieth birthday, January 8.

COLONEL I. H. WING, of Batfield, Wisconsin, an alumnus of Bowdoin College, has given the college $50,000 to endow the chair of mathematics.

THE Royal astronomical society has awarded its gold medal to Professor W. N. CAMPBELL, of the Lick Observatory, for his researches on the motion of stars in the direction of the line of sight.

DR. E. B. WILSON has been promoted to an assistant professorship of mathematics at Yale University.

PROFESSOR W. J. HUSSEY, of Lick Observatory, has been appointed professor of astronomy at the University of Michigan.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AMES (L. D.). An arithmetic treatment of some problems in analysis situs. (Diss., Harvard.) Baltimore, Friedenwald, 1905. 4to. (*American Journal of Mathematics*, Vol. 27, pp. 343–380.)

ANGUS (A. H.). A preliminary course in differential and integral calculus. London, Longmans, 1906. 8vo. 116 pp. Cloth. 2s. 6d.

DÖLP (H.). Grundzüge und Aufgaben der Differential- und Integralrechnung nebst den Resultaten. Neubearbeitet von E. Netto. 11te Auflage. Giessen, Töpelmann, 1905. 8vo. 4 + 216 pp. Cloth. M. 1.80

DUCCI (E.). Sezioni del cono retto circolare e deduzione delle loro principali proprietà; lezioni. Melfi, Grieco, 1905. 8vo. 49 pp.

FIELD (P.). On the form of a plane quintic curve with five cusps. 1906. 4to. (*Transactions of the American Mathematical Society*, Vol. 6, pp. 26–32.)

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